(De)stabilizing speculation on futures markets
An alternative view point*

Roger Guesnerie
Delta (joint research unit CNRS-EHESS-ENS), Paris, France

Jean-Charles Rochet
GREMAQ and IDEI, Université des Sciences Sociales, Toulouse, France

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This paper offers a new interpretation for possible destabilizing effects of opening futures markets. In our model, adapted from Stein (1987), speculation on futures markets reduces the likelihood of occurrence of a Rational Expectations Equilibrium. Although the equilibrium price is less volatile after the futures market is opened (which is usually viewed as a stabilizing effect), traders may find it more difficult or even impossible to coordinate their expectations in order to implement this equilibrium.

1. Introduction

There seems to be a widespread opinion among practitioners that opening futures markets can (at least in some cases) have destabilizing effects. Indeed several empirical studies [like for instance Figlewski (1981), Simpson and Ireland (1985)] confirm that volatility on the spot market tends to increase after a futures market is introduced. However, theorists have so far found it difficult to design models in which such a phenomenon occurs: Turnovsky (1979, 1983), Turnovsky and Campbell (1985) (to quote only a few recent works) unambiguously conclude that futures markets should always have stabilizing effects. As remarked by Friedman, this is consistent with the fact that speculators tend to buy when prices are low and to sell when prices are high.

Only a few theoretical papers obtain possible destabilization, either because opening futures markets induce producers to take more risks

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This paper is an attempt to reconcile the theorists' inclination to stress the stabilizing effect of speculation with the practitioners' contrary feelings. We do that, however, by giving to the word stabilization two different meanings. On the one hand, in line with the teaching of most theoretical models, speculation has a stabilizing effect in the sense of reducing the variance of rational expectations equilibrium prices; on the other hand, speculation reduces the likelihood of the occurrence of a rational expectations equilibrium. This latter fact can be viewed as destabilizing, in the sense that it destabilizes expectations and makes the system less predictable.\(^1\)

Our evaluation of the likelihood or stability of rational expectations follows a line of research initiated by Guesnerie (1988): we study the stability of mental ('eductive') coordination strategies. Our approach is different from studying the ('evolutive') stability of a learning process, but is more akin to the rationalizability concept of Bernheim (1984) and Pearce (1984) in noncooperative Game Theory.

The paper provides an example of a model, a simplified version of Stein (1987), in which speculation is both, according to the precise sense given to the word, stabilizing and destabilizing: Although the equilibrium price is less volatile after the futures market is opened, traders may find it more difficult or even impossible to coordinate their expectations in order to implement this equilibrium. Then, the opening of new markets threatens in some sense the occurrence (stability) of the new (rational expectations) equilibrium.

Note that this study does not reject the concept of Rational Expectations Equilibrium but supplements it with a stability criterion. This criterion, however, suggests that there are contexts in which, even though there is a unique Rational Expectations Equilibrium, other outcomes may be equally likely because they can be rationalized by reasonable conjectures. Note also that, although our model is very specific, we believe that the point of view we defend has a general relevance: A consequence of creating new markets or new financial instruments may be that the necessary coordination between agents gets slower or more difficult.

The model is presented in section 2. The rest of the paper consists of an analysis of the problems under consideration that remains (hopefully) intuitive. Section 3 studies the case without a futures market, while section 4 is dedicated to the situation when a futures market has been opened. Section 5 discusses several lines of possible extension.

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\(^1\)As the reader will notice later, we do not argue, however, that destabilization in this sense increases the variance of prices, for the simple reason that we do not present any 'positive' theory that would be an alternative in this destabilization case to the rational expectations theory.
2. The basic framework

The model is inspired by Stein (1987).

There are two dates in the economy ($t = 1, 2$) and two goods: wheat and an aggregate consumption good taken as a numéraire. The demand for wheat from the consumers sector is described by simple aggregate demand functions for wheat:

$$P_t = \max(0, k(d_t - D_t)), \quad t = 1, 2,$$

where $d_1, d_2, k$ are positive constants, $D_t$ denotes the demand for wheat at date $t$, and $P_t$ denotes the price of wheat at date $t$.

There is no explicit modelling of production: crops endowments are only supposed to be random variables $\tilde{o}_1, \tilde{o}_2$. We follow the usual convention that tilde symbols indicate random variables. For the sake of simplicity it is assumed that these stochastic endowments are independently distributed and that both distributions have the same variance $\sigma^2$. Their mean values are denoted $\bar{o}_1, \bar{o}_2$.

Following Stein (1987), we introduce two categories of traders:

- 'primary' or 'spot' traders, who can store inventories,
- 'secondary' traders who cannot store inventories.

All spot traders have the same inventory cost function, which is supposed to be quadratic:

$$C(x) = \frac{1}{2} Cx^2.$$

In order to stick to the idea that each trader has negligible market power, we formalize in the first part the trader sector as a continuum of traders of each type: the set of spot traders is denoted $(I, \mu)$ and that of secondary traders is denoted $(J, v)$. The total masses are

$$\mu(I) = N, \quad v(J) = M.$$

Now we are going to present the different market structures under consideration.

**Case 1** is the case without futures markets. Only spot traders are active and the timing of observations and decisions is the following: first, traders observe $\omega_1$, the realization of $\tilde{o}_1$, second, they make an inventory decision; third, given inventory decisions which are not publicly observed, the market for wheat clears at a market price that is publicly observed. Finally, the

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2Without a quadratic cost and a linear demand, our stability analysis would only be true locally.

3A discussion of the case with a set of (non-negligible) traders will be presented in section 5.
second period crops and the second period market price are revealed. The important assumption is that the inventory decisions cannot be made conditional on the first period market clearing price.

Case 2 is the case with futures markets: first, all traders determine their positions on the futures market as a function of their anticipations of spot prices and the futures market clears. Then the story follows as in Case 1.

In order to complete the description of the model, we need to define preference. Again we adopt a simple assumption, i.e., that all traders have identical preferences of the mean–variance type:

$$V(i) = E(\pi(i)) - \frac{1}{2} B \text{var}(\pi(i)),$$

where $\pi$ denotes the trader's profit and $B$ is a positive constant reflecting the trader's risk aversion. Finally, the interest rate is assumed to be zero.

Let us now analyse equilibrium in both situations.

3. Without a futures market

We first describe traders' behaviour and the rational expectations equilibrium. We then discuss the 'eductive' stability of the rational expectations equilibrium that determines whether the equilibrium is (or is not) strongly rational [see Guesnerie (1988)].

3.1. Traders' behaviour and rational expectations equilibrium

Only spot traders are active. With the structure of information under consideration, their strategy will depend upon their expectations of the spot prices. Let us call $P^*_1(i)$ and $P^*_2(i)$ trader $i$'s expectations on the price of wheat at periods 1 and 2 (we consider here only point expectations). We have

$$P^*_1(i) = k(d_1 + X^c(i) - \omega_1),$$

$$P^*_2(i) = k(d_2 - X^c(i) + \bar{\omega}_2),$$

where $X^c(i)$ denotes the expectation by trader $i$ of the total level of inventories held by other traders.

Note that $P^*_2(i)$ is random, but that the quantity $X^c(i)$ does not affect the variance of this random variable:

$$\text{var}(\tilde{P}^*_2(i)) = k^2 \sigma^2.$$

The anticipated profit of trader $i$ is
\[\pi(i) = (\bar{P}_2(i) - P_1^s(i))x(i) - \frac{1}{2}Cx^2(i), \quad (3)\]

and his utility is
\[V(i) = E(\pi(i)) - \frac{1}{2}B \text{var}(\pi(i)),\]
or, using (1), (2) and (3),
\[V(i) = k(X_0 - 2X^*(i))x(i) - \frac{1}{2}(Bk^2\sigma^2 + C)x^2(i),\]
where
\[X_0 = d_2 - \bar{d}_2 - d_1 + \omega_1. \quad (4)\]

The optimal strategy of trader \(i\) is thus
\[x^*(i) = \frac{k(X_0 - 2X^*(i))}{Bk^2\sigma^2 + C},\]
or putting
\[t = \frac{k}{Bk^2\sigma^2 + C}, \quad (4')\]
\[x^*(i) = t(X_0 - 2X^*(i)). \quad (5)\]

**In a Rational Expectations Equilibrium**, traders use their optimal strategies and expectations are fulfilled: i.e.
\[X^*(i) = X^* = \int x^*(i') \, d\mu(i') \quad \text{for all } i \text{ in } I.\]

Using eq. (5) and integrating over \(i\), we obtain
\[X^* = \frac{Nt}{1 + 2Nt} X_0\]
or
\[X^* = \frac{X_0}{2 + C/kN + Bk\sigma^2/N}, \quad (6)\]

where parameters \(X_0\) and \(t\) were defined in eqs. (4) and (4').

The computation is straightforward. The reader will check that the comparative statics analysis of this equilibrium with respect to \(C, B,\)

\[\text{Note that } X_0 \text{ is random but known at the decision time, when } \omega_1 \text{ is realized. For the sake of simplicity, positivity constraints are ignored in our informal analysis: we will assume that } X_0 > 0 \text{ with probability one.}\]
fits economic intuition: the equilibrium level of inventories increases with $X_0$ and decreases when the cost coefficient $C$, the risk aversion coefficient $B$, the variance $\sigma^2$ and the sensitivity of prices to supply conditions $(k)$ increase. Note also that an increase in $N$ – that indeed decreases the total social cost of inventories – has an effect similar to that of a decrease of $C$.

3.2. Eductive stability of the rational expectations equilibrium

As explained in the introduction, we want to focus our attention on the mental coordination process through which traders gradually adjust their expectations $X^e(i)$. Following Guesnerie (1988), we study a process of a kind of 'collective introspection' that would support the above equilibrium and that is reminiscent of rationalizability in the sense of Bernheim (1984) and Pearce (1984): Traders form their expectations by successively eliminating those expectations which could not stem from rational behaviour of their competitors.

In a first step, strategies that can never be optimal are eliminated. Since inventories $X^e(i)$ are non-negative, eq. (5) implies

$$0 \leq x^e(i') \leq tX_0.$$  

This condition being known by everybody, individual expectations must be such that

$$0 \leq X^e(i') = \int x^e(i'') \, d\mu(i'') \leq NtX_0.$$  

Then, after one step, any spot trader's expectation on total inventories will be restricted to

$$D_1 = [0, NtX_0].$$

But in turn, these restrictions have consequences on individual strategies, and may or may not induce further restrictions on expectations. To see that, let us compute the expectation formed by agent $i$ on the inventory decision of agent $i'$, which we denote $x^e(i,i')$. Using again eq. (5) we obtain

$$x^e(i,i') = t(X_0 - 2X_{i,i'}^e),$$

where $X_{i,i'}^e$ is the expectation formed by agent $i$ on the expectation by agent $i'$ of total inventories. Integrating over $i'$ we obtain

$$X^e(i) = Nt(X_0 - 2\bar{x}^e(i)),$$

where $\bar{x}^e(i)$ denotes the expectation formed by $i$ on the average expectation
of other traders. Denoting by $\Phi(\hat{x}^n(i))$ the right-hand side of eq. (7), we can deduce that $i$'s expectation, that was restricted to $D_1$ after the first step, also belongs after the second step to $\Phi(D_1)$ and hence to $D_1 \cap \Phi(D_1)$. Also, we can now define by induction the set of possible expectations after $n+1$ steps of elimination:

$$D^{n+1} = D^n \cap \Phi(D^n),$$

with

$$\Phi(x) = Nt(X_0 - 2x)$$

and

$$D^1 = [0, NtX_0].$$

Our stability concept reflects the convergence of the mental process just described:

**Definition 1.** The Rational Expectations Equilibrium above is Strongly Rational (or stable in the eductive sense) if and only if:

$$\bigcap_{n} D^n = \{X^*\}.$$  

Fig. 1 visualizes the graph of $\Phi(x)$ and the first bissectrix and then, along the

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Note that we assume that traders do not only know the system but also know that the others know and know that (the others know) that the others know. … Finally, we will exploit the full power of a Common Knowledge Assumption.
x-axis, $D_1$ and then $D_2 = D_1 \cap \Phi(D_1)$, etc. ... Elementary algebra immediately confirms what fig. 1 suggests.

**Proposition 1.** A necessary and sufficient condition for the above equilibrium to be strongly rational is

$$N < N^* = \frac{Bk^2\sigma^2 + C}{2k} = \frac{1}{2\tau^2}.$$  

Consider then successively the following coefficient changes:

- there are more traders ($N$ increases);
- these traders are less risk averse ($B$ decreases);
- the crops are less uncertain ($\sigma$ decreases);
- inventories become less costly ($C$ decreases).

It is noticeable that these coefficient changes, ceteris paribus, have a negative effect on eductive stability while they have a positive effect on the rational expectations equilibrium inventory level: this is not surprising since these coefficient changes that increase the magnitude of the inventory decision also make it more difficult to 'predict'. Note, however, that $X_0$, which is a key factor in the explanation of the size of inventories, does not affect eductive stability.

4. With a futures market

4.1. Traders' behaviour and rational expectations equilibrium

In this new context, let us analyse in turn the strategies of primary traders and then of secondary traders. Remember our assumptions on the timing of decisions: first all traders determine their positions on the futures market as a function of their anticipations of spot prices and this market clears at price $P_t$. Then the story parallels that of section 3: secondary traders have nothing left to decide, while primary traders choose their inventories (before knowing the period 1 spot price) as a function of $P_t$ and their anticipations of spot prices.

Consider now the decision problem of primary trader $i$: he has to submit a demand schedule $f = f(i, P_t)$ to the futures market and to choose an inventory level $x = x(i, P_t)$ after having observed the clearing price $P_t$ on the futures market. For given spot price expectations $P^*_1(i)$ and $P^*_2(i)$, his anticipated profit is

$$\Pi(i) = [\hat{P}^*_2(i) - P_t]f + [\hat{P}^*_2(i) - P^*_1(i)]x - \frac{1}{2}Cx^2.$$  

$\phi$ is contracting if and only if $2N\tau < 1$. 

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6. Elementary algebra isimmediately confirms what fig. 1 suggests.
Then we compute his utility:

\[ V(i) = [E(\bar{P}_2(i)) - P_t]f + [E(\bar{P}_2'(i)) - P_1'(i)]x - \frac{1}{2}Cx^2 - \frac{1}{2}B \text{var}(\bar{P}_2(i))(x + f)^2. \] (9)

Taking derivatives with respect to \(f\) and \(x\), we obtain the first-order conditions:

\[ E[\bar{P}_2'(i)] - P_t - B \text{var}(\bar{P}_2'(i))(x + f) = 0 \] (10)

and

\[ E[\bar{P}_2'(i)] - P_1'(i) - B \text{var}(\bar{P}_2'(i))(x + f) - Cx = 0. \] (11)

This implies

\[ f = f(i, P_t) = \frac{P_t - P_1'(i)}{C} + \frac{E[\bar{P}_2'(i)] - P_t}{B \text{var}(\bar{P}_2'(i))}, \] (12)

and

\[ x = x(i, P_t) = \frac{P_t - P_1'(i)}{C}. \] (13)

The decision of secondary trader \(j\) only concerns his position \(f = f(j, P_t)\) on the futures market. Noting that his problem is a variant of the primary trader's problem (obtained by adding the constraint \(x = 0\)) we can use eq. (10) with \(x = 0\), to get

\[ f(j, P_t) = \frac{E[\bar{P}_2'(j)] - P_t}{B \text{var}(\bar{P}_2'(j))}. \] (14)

Let us consider, as an interesting benchmark, the case where traders have homogenous expectations, denoted \(P_1^e\) and \(\bar{P}_2^e\) (dropping \(i\) or \(j\) indices).

Using (12) and (14) we obtain that the clearing price on the futures market is a convex combination of these spot price expectations:

\[ P_t = \alpha E[\bar{P}_2^e] + (1 - \alpha)P_1^e. \] (15)

with

\[ ^7\text{Note that these formulas reflect the hedging behaviour of primary traders who can sell forward their inventories through the futures market. Notice also that eq. (13) is the standard result that the future price is the action certainty equivalent price.} \]

\[ ^8\text{This formula is compatible with the economic intuition that 'in equilibrium' the futures price should be somewhere in between the (expected) first-period price and the (expected) second-period price.} \]
\[ \alpha = \frac{(N + M)/Bk^2 \sigma^2}{N/C + (N + M)/Bk^2 \sigma^2}. \]  

(16)

Total inventories are easily obtained as well, by integrating (13) and using (15) above:

\[ X = \int x(i, P_t) \, d\mu(i) = \frac{\alpha N}{C} \left[ \mathbb{E}[\bar{P}_t] - P_t^* \right]. \]

The final step is to express anticipated spot prices as functions of expected inventories \(X^e\):

\[
\begin{align*}
\{ P_t^* &= k[d_1 - \omega_1 + X^e] \\
\mathbb{E}[\bar{P}_t^*] &= k[d_2 - \bar{\omega}_2 - X^e] 
\end{align*}
\]

We obtain the relation between actual (total) inventories \(X\) and expected (total) inventories \(X^e\):

\[ X = \frac{\alpha k N}{C} \left[ X_0 - 2X^e \right]. \]  

(17)

In particular the (unique) Rational Expectations Equilibrium is obtained for \(X = X^e = X^{**} = \frac{X_0}{2 + C/\alpha k N}\)

(18)

where \(X_0\) and \(\alpha\) are given by eqs. (4) and (16). A straightforward computation gives the open interest on the futures market:

\[ F^{**} = \int f(j, P_t) \, dv(j) = \frac{M}{N + M} X^{**}. \]

Rewriting \(X^{**}\) as

\[ X^{**} = \lambda(k, C, B, \sigma, N, M) X_0 \]  

(19)

with

\[ \frac{1}{\lambda(\cdot)} = 2 + \frac{C}{kN} + \frac{Bk \sigma^2}{N + M}. \]

one can check that for \(M = 0\), it coincides unsurprisingly with the equilibrium

\[ \text{As in the first case, the reader is exhorted to check briefly the comparative statics of the rational expectations equilibrium as a function of the parameters } N, M, \sigma \text{ [formula (18)].} \]
level of inventories in the previous model without futures. It can thus easily be shown that the variance of \( P_1 \) [i.e. \( k^2((1-\lambda(\cdot))^2 \text{var} X_0) \)] is unambiguously decreased by the opening of the futures market. This is the announced stabilizing property of futures markets.

4.2. The eductive stability of equilibrium

The eductive process to be examined now is more complicated than previously. For primary trader \( i \), decisions consist of a demand schedule \( f(i, \cdot) \) to be submitted to the futures market's 'auctioneer' and of an inventory strategy \( x(i, \cdot) \), conditional on the futures market clearing price. Similarly, for secondary trader \( j \), decision consists of choosing a demand schedule \( f(j, \cdot) \). Thus, a priori, these decisions belong to an infinite dimensional space and mental coordination seems more difficult to achieve. However, because of our simplifying assumptions, these strategies will in fact be parameterized by a small number of variables. The reasoning is as follows: consider the optimal decisions of a primary trader \( i \) having spot price expectations \( \hat{P}_1(i) \) and \( \hat{P}_2(i) \). These optimal decisions are given by conditions (12) and (13):

\[
\begin{align*}
f(i, P_t) &= \frac{E[\hat{P}_2(i)]}{B \text{var}(\hat{P}_2(i))} + \frac{P_1^*(i)}{C} - P_t \left[ C + \frac{1}{B \text{var}(\hat{P}_1(i))} \right], \\
x(i, P_t) &= \frac{P_t - P_1^*(i)}{C}.
\end{align*}
\]

Using market clearing equations (2) and (3) and taking into account the expectations on inventories this can also be written as

\[
\begin{align*}
f(i, P_t) &= d_2 - \hat{\omega}_2 - X^e(i) + \frac{k}{C} \left[ d_1 - \omega_1 + X^e(i) \right] - P_t \left[ \frac{1}{C} + \frac{1}{Bk^2 \sigma^2} \right], \\
x(i, P_t) &= \frac{P_t - k}{C} \left[ d_1 - \omega_1 + X^e(i) \right].
\end{align*}
\]

\(^{10}\)In the particular case when \( C \) equals zero, as already remarked by Stein (1987), opening a futures market is just like increasing the number of spot traders to \( N + M \). [Formula (19) makes clear the mechanism underlying this fact.]

\(^{11}\)As above, we provide here an intuitive version of a more rigorous game theoretical rationalizability argument.
Thus as expectations vary these demand schedules all remain linear (in \( P_t \)) with the same slopes: they only differ by their intercepts. They can even be parameterized by a one-dimensional variable, namely \( X^e(i) \), the anticipation by \( i \) of total inventories.

A similar reasoning applies to agent \( j \), for whom eqs. (14) and (3) give

\[
f(j, P_t) = \frac{d_2 - \bar{\omega}_2 - X^e(j)}{Bk\sigma^2} - \frac{P_t}{Bk^2\sigma^2}.
\]

(22)

Let us try now to determine how these anticipations on inventories are formed: the first step is to solve for \( P_t \). Integrating (20) over \( i \) and (22) over \( j \), adding up and rearranging terms, we obtain

\[
P_t = \alpha k \left[ d_2 - \bar{\omega}_2 - \frac{N}{N+M} \bar{X} - \frac{M}{N+M} \bar{\bar{X}} \right]
\]

\[
+ (1 - \alpha) k \left[ d_1 - \bar{\omega}_1 + \bar{X}_t \right],
\]

(23)

where

\[
\bar{X} = \frac{1}{N} \int X^e(i) \, d\mu(i) \quad \text{and} \quad \bar{\bar{X}} = \frac{1}{M} \int X^e(j) \, dv(j)
\]

denote respectively the average expectations of total inventories by primary and secondary traders.

We can now derive the expectation made by agent \( i \) of the inventory that will be chosen by agent \( i' \), which we denote \( x^e(i,i') \). For that purpose we first compute the expectation of \( P_t \) by agent \( i \) [using eq. (23)]:

\[
P_t^i = \alpha k \left[ d_2 - \bar{\omega}_2 - \frac{N}{N+M} \bar{X}^e_i - \frac{M}{N+M} \bar{\bar{X}}^e_i \right]
\]

\[
+ (1 - \alpha) k \left[ d_1 - \bar{\omega}_1 + \bar{X}_t^i \right],
\]

(24)

where \( \bar{X}^e_i \) and \( \bar{\bar{X}}^e_i \) denote respectively the expectations by agent \( i \) of \( \bar{X} \) and \( \bar{\bar{X}} \). That is, \( \bar{X}^e_i \) and \( \bar{\bar{X}}^e_i \) represent the expectations made by \( i \) of the average expectations of total inventories by primary and secondary traders. Finally, using eq. (21) we obtain

\[
x^e(i,i') = \frac{P_t^i}{C} - \frac{k}{C} \left[ d_1 - \bar{\omega}_1 + X^e_{i,i'} \right],
\]

where \( X^e_{i,i'} \) denotes the expectation by agent \( i \) of the expectation by agent \( i' \) of
total inventories. Replacing $P_i^t$ by its value given in eq. (24) we obtain, after
simple manipulations,

$$x^e(i,i') = \frac{k}{C} \left[ d_2 - \bar{\omega}_2 - \frac{N}{N+M} \bar{X}_i^e - \frac{M}{N+M} \bar{X}_i^e \right]$$

$$+ \frac{(1-\alpha)k}{C} \left[ d_1 - \omega_1 + \bar{X}_i^e \right] - \frac{k}{C} \left[ d_1 - \omega_1 + X_{i,i'}^{ee} \right]$$

$$= \frac{\alpha k}{C} \left[ X_0 - \frac{N}{N+M} \bar{X}_i^e - \frac{M}{N+M} \bar{X}_i^e \right]$$

$$+ \frac{(1-\alpha)k}{C} \bar{X}_i^e - \frac{k}{C} X_{i,i'}^{ee},$$

or

$$x^e(i,i') = \frac{k}{C} \left[ \alpha X_0 \left( \frac{\alpha N}{N+M} - (1-\alpha) \right) \bar{X}_i^e - \frac{\alpha M}{N+M} \bar{X}_i^e - X_{i,i'}^{ee} \right].$$

Now integrating over $i'$, we obtain agent $i$'s expectations of total inventories:

$$X^e(i) = \frac{Nk}{C} \left[ \alpha X_0 \left( \frac{\alpha N}{N+M} - (1-\alpha) \right) \bar{X}_i^e - \frac{\alpha M}{N+M} \bar{X}_i^e - \bar{X}_i^e \right],$$

where we have used the fact that, by linearity of expectations,

$$\bar{X}_i^e = \frac{1}{N} \int \bar{X}_{i,i'}^e \, d\mu(i').$$

That is to say: the expectations by $i$ of the average expectation of primary traders is the average of the expectation by $i$ of individual expectations of primary traders $i'$.

Finally, we obtain the expression relating $X^e(i)$, agent $i$'s expectation on total inventories, to his expectations on the average expectations of primary traders, $\bar{X}_i^e$ and secondary traders, $\bar{X}_i^e$:

$$X^e(i) = \frac{\alpha Nk}{C} \left[ X_0 - \frac{2N+M}{N+M} \bar{X}_i^e - \frac{M}{N+M} \bar{X}_i^e \right]. \quad (25)$$

Let us remark in passing that the value of the (unique) Rational Expectations Equilibrium $X^{**}$ is deduced immediately from (25) by requiring homogeneous expectations:

$$X^e(i) = \bar{X}_i^e = \bar{X}_i^e = \frac{X_0}{2+C/\alpha k N} = X^{**}.$$
We are now in a position to study the convergence of the eductive process described above. Using the same method as in section 3, and similar notations, we can define

$$D^\dagger_F = \left[ 0, \alpha \frac{NK}{C} X_0 \right].$$

This is the set of possible expectations after one step of elimination (simply using the fact that inventories are non-negative). The subscript \( F \) refers to the existence of the futures market. Moreover condition (25) can be rewritten as

$$X^e(i) = \Phi_F (\bar{X}^e(i), \bar{X}^e(i)),$$

where by definition

$$\Phi_F(x, y) = \alpha \frac{NK}{C} \left[ X_0 - \frac{2N + M}{N + M} x - \frac{M}{N + M} y \right]$$

describes the agents' mental adjustment process when there is a futures market. As in the previous section the set of possible expectations after \((n+1)\) steps of elimination is defined by induction:

$$D^{n+1}_F = D^n_F \cap \Phi_F (D^n_F \times D^n_F).$$

A straightforward argument, analogous to the one of the previous section, shows that eductive stability is obtained if and only if \( 2\alpha NK/C \) is less than 1. Replacing \( \alpha \) by its expression given in eq. (16), we obtain

**Proposition 2.** With a futures market, the Rational Expectations Equilibrium is strongly rational if and only if

$$\frac{C}{N} + \frac{Bk^2 \sigma^2}{N + M} > 2k.$$  \hspace{1cm} (26)

Now consider the left-hand side of condition (26): it is a decreasing function of \( N \) and \( M \), equal to \( +\infty \) when \( N \) equals 0. Moreover its right-hand side is equal to \( (C + Bk^2 \sigma^2)/N^* \). Therefore, when \( M \) equals 0, condition (26) reduces to \( N < N^* \). For positive \( M \), it is equivalent to \( N < N^*(M) \), where \( N^*(\cdot) \) is a decreasing function. Thus we have proved

\(^{12}\)Again, this condition coincides with the one of Proposition 1 in the case \( M = 0 \) (this reflects the fact that in this case all agents realize that the futures market is inactive, so that we are back to the preceding case). Note that in both sections, the eductive stability conditions are equivalent to the fact that the equilibrium value of inventories is smaller than \( X_0/4 \). Note also that in this setting \( M \) is better interpreted on the intensity of the speculation activity rather than the thickness of the market (our model does not incorporate the ingredients relevant to the analysis of what is behind the 'thickness' phenomenon.
Proposition 3. - For every $M \geq 0$, there exists a constant $N^*(M)$ such that eductive stability is equivalent to

$$N < N^*(M).$$

- $N^*(\cdot)$ is a decreasing function of $M$ and $N^*(0) = N^*$.
- As a consequence, eductive stability of the Rational Expectations Equilibrium is less likely with a futures market ($N < N^*(M)$) than without ($N < N^*$).

Proposition 3 expresses in a simple way the fact that the set of exogenous parameters (i.e. $k, B, \sigma, C, N, M$) of our model for which the equilibrium is strongly rational is reduced when the potential 'size' of the futures market, as measured by $M$, increases.

Let us summarize now the stabilizing or destabilizing consequences of opening a futures market in our very simple model:

- Once agents have been convinced that all other agents will behave in accord with the Rational Expectations Hypothesis, there is a unique equilibrium in both cases (before and after opening a futures market). Moreover, the unconditional variance of spot prices is smaller in the second case. In other words, if we compare the properties of the Rational Expectations Equilibria in terms of price variability, opening a futures market has stabilizing consequences.

- However, opening the futures market has also the consequence that the eductive process (through which agents get gradually convinced to use their equilibrium strategies) is now slower.\(^{13}\) In some cases ($N(M^*) < N < N^*$) introducing the futures market may even destroy the stability of this process.\(^{14}\)

At this stage, the implications of our findings are open to different interpretations. One has just been sketched in footnote 14. Let us present another one: one may conjecture that if exogenous parameters are not too variable (for instance if $\tilde{w}_t$ is stationary, and $N, M$ are constant...), the unique Rational Expectations Equilibrium is an appealing focal point that

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13 As the careful reader will have checked by looking at eq. (25).
14Again we do not argue that in the unstable case the volatility of observed prices will be greater since in this case, if one accepts the logic of our position, one cannot refer to any well defined 'positive' theory that would explain prices. Note, however, that a number of stories based on the rationalizability concepts would suggest such a greater volatility. For example, when the equilibrium is not eductively stable, the set of rationalizable solutions of the model necessarily contains a small symmetric interval around the rational expectations equilibrium; if the outcome is picked at random in this interval the corresponding rationalizable price is a noisy estimate of the rational expectations prices and hence has higher variance. In fact the set of rationalizable solutions is much larger – it is $D^k$ – so that the just sketched argument for higher volatility can be made much stronger.
will ultimately prevail. However, if the environment is changing, and in any case in the transitory phase that follows the opening of the futures market, the consequence of eductive instability is that many outcomes other than the unique Rational Expectations Equilibrium can as well be rationalized in the sense of Bernheim and Pearce. Rather than rejecting the concept of Rational Expectations Equilibrium, this argument, by stressing that many other outcomes are equally reasonable, at least transitorily, tends to qualify its universal validity.

5. Further discussion

There are a large number of possible directions for further research. We will briefly sketch some of them.

5.1. Relaxing the continuum assumption

Instead of having non-atomic sets of traders, one could consider discrete sets \{1, ..., N\} of spot traders, and \{1, ..., M\} of secondary traders, and leave the rest of the model unchanged. The main difference with the previous case is that individual agents would now have some market power. The Rational Expectations Equilibrium is now a Cournot-Nash Equilibrium. In the case without a futures market, 'eductive' stability can be shown to be equivalent to the stability of the Cournot-Nash tâtonnement (see the comments below). Straightforward but tedious computations show that this corresponds to the case when

\[ N < N^* + 3. \]

In the futures market case, the introduction of such individual market powers complicates the algebraic manipulations in such a significant way that the comparison with the previous setting becomes messy. However, we conjecture that the same kind of conclusions can be obtained.

5.2. 'Eductive' versus 'adaptive' learning

Following early work on learning, the literature has often assumed that agents use simple learning rules (for example adaptive) in order to update their expectations. The use of such learning rules would lead to evolution which takes place in real time, in contrast to the virtual time associated to eductive learning. It is natural to ask how adaptive learning rules perform, in comparison with eductive rules, in terms of convergence to rational expectations equilibria.

Consider then the model of section 3 (without a futures market) and
assume that all spot traders have the same expectations (it is denoted $X^e_t$) on inventories at the $t$th repetition of the static game of section 3 and that these expectations are revised as follows:

$$X^e_{t+1} = aX^e_t + (1-a)X_t \quad (0 < a \leq 1)$$

(27)

where $X_t$ is the inventory realization of period $t$.

Using (5), we find

$$X_t = \frac{kN(X_0 - 2X^e_t)}{Bk^2\sigma^2 + C}$$

and

$$X^e_{t+1} = \frac{(1-a)kNX_0}{Bk^2\sigma^2 + C} + \left( a - \frac{2(1-a)kN}{Bk^2\sigma^2 + C}\right)X^e_t.$$ 

(28)

Convergence of (28) obtains if and only if

$$-1 < a - \frac{2(1-a)kN}{Bk^2\sigma^2 + C} < 1$$

i.e.

$$\frac{1+a}{1-a} > 2Nt.$$ 

(29)

This formula shows that for a given specific value of $a$, the convergence condition (29) generally differs from the convergence condition (8) of section 3. However, if one requires that convergence obtains for every value of $a$ in $]0,1[ - as it may seem natural since $a$ is an ad hoc coefficient - we have to require that $2Nt < 1$, which is exactly condition (8).

Thus, we find that the property obtains when $N < N^*$. The equivalence between eductive and adaptive stability in the sense mentioned above confirms similar statements made previously in different contexts [Gabay and Moulin (1980), Guesnerie (1988)]. It can be shown to hold also in our model of section 4 (with a futures market). However, it is not clear that this equivalence would remain true if we changed the timing of observations and decisions, which we discuss now.

5.3. The timing of observations and decisions

One can wonder whether our results are robust to a change in the timing of observations and decisions. The answer to this question seems delicate. On the one hand, prices might now transmit information and make the
coordination of expectations significantly easier. On the other hand, it is not clear at all whether the equilibria that transmit information through prices can be strongly rational, so that the transmission issue may be irrelevant to our problems. A full assessment of these problems requires another study. Consider for instance the case where agents can submit demand functions on the spot market. When there is no futures market, the stability problems of the rational expectations equilibrium disappear. The optimal inventory decision of each agent can be conditioned on \( P_1 \), which completely reveals \( X \).

Given that in equilibrium one has necessarily \( P_1 = k(d_1 + X - \omega_1) \), the utility of \( i \) can be written

\[
V_i = (k[d_2 - \bar{\omega}_2 + d_1 - \omega_1] - 2P_1)x_i - \frac{1}{2}(C + Bk^2\sigma^2)x_i^2.
\]

It is maximum for

\[
x_i = \frac{k[d_2 - \bar{\omega}_2 + d_1 - \omega_1] - 2P_1}{C + Bk^2\sigma^2},
\]

which is a dominant strategy for agent \( i \). Therefore the unique REE is rationalized after one step of elimination of dominated strategies.

When there is a futures market, things are more complex. The utility function of agent \( i \) writes

\[
V_i = (E(\bar{P}_2) - P_1)f_i + (E(\bar{P}_2) - P_1)x_i - \frac{1}{2}C_x^2 - \frac{1}{2}B \text{var}(\bar{P}_2)(x_i + f_i)^2.
\]

Suppose in a first step that all positions on the futures market (and therefore \( P_1 \)) are given. Using the same reasoning as above, we can determine agent \( i \)'s optimal demand schedule on the spot market:

\[
x_i(P_1) = \frac{k[d_2 - \bar{\omega}_2 + d_1 - \omega_1] - Bk^2\sigma^2f_i - 2P_1}{C + Bk^2\sigma^2}.
\]

Therefore the equilibrium price on the spot market can be determined:

\[
P_1 = k[(1 - \gamma)(d_1 - \omega_1) + \gamma(d_2 - \bar{\omega}_2 + kB\sigma^2F)],
\]

where

\[
\gamma = \frac{Nk}{C + Bk^2\sigma^2 + 2Nk}
\]

and
R. Guesnerie and J.-C. Rochet, (De)stabilizing speculation on futures markets

\[ F = -\frac{1}{N} \int f_i \, d\mu(i) \]

is the average position of spot traders on the futures market. The total amount of inventories can then be expressed as a function of \( F \):

\[ X(F) = \gamma [X_0 + kB\sigma^2 F] \]

In a second step, each agent has to determine his position on the futures market as a function of \( P_t \) and its expectations on \( F \). After straightforward computations we obtain

\[
\begin{align*}
\hat{f}_t^i &= \frac{d_2 - \bar{\omega}_2}{Bk\sigma^2} + \frac{k}{C} \left( d_1 - \omega_1 \right) - \left( \frac{C - Bk^2\sigma^2}{Bk\sigma^2 C} \right) \gamma X_0 \\
&\quad - \frac{C - Bk^2\sigma^2}{Bk^2\sigma^2 C} P_t - \left( \frac{C - Bk^2\sigma^2}{C} \right) \gamma F_i
\end{align*}
\]

for primary trader \( i \) (where \( F_i \) denotes the expectation formed by \( i \) on \( F \)), and

\[
\hat{f}_t^j = \frac{d_2 - \bar{\omega}_2}{Bk\sigma^2} - \gamma X_0 - \gamma F_j - \frac{P_t}{Bk^2\sigma^2}
\]

for secondary trader \( j \). Using the same line of reasoning as in section 4 we can therefore compute the equilibrium price and the open interest on the futures market as a function of

\[ F = M \left[ \beta \gamma \hat{F} - \delta \alpha \hat{F} \right] \]

where \( K \) is a constant and

\[ \alpha = \frac{1}{Bk^2\sigma^2}, \quad \beta = \frac{1}{C} + \alpha, \quad \delta = \gamma \left( 2 - \frac{\beta}{\alpha} \right). \]
Eductive stability is obtained if and only if
\[ \alpha N + \beta M > M(\alpha |\delta| + \beta \gamma), \]
which is always true if \( \delta \) is positive. However, when \( \delta \) is negative, this gives a restriction on \( N \) and \( M \):
\[ \frac{C\alpha}{N} + \frac{1 + \alpha C}{M} > \frac{1}{N + (C + 1/\alpha)/2k}. \]

We will not provide a complete discussion of the latter inequality, the analysis of which is more complex than the analysis of the previous corresponding formula (26) [in particular the shape of the set of \((M, N)\) for which the inequality is met is more complex than previously]. However, the general flavour of the findings is the same: opening futures markets significantly restricts the set of exogenous parameters for which the model has a strongly rational expectations equilibrium.

5.4. Relaxing some simplifying assumptions of the present model

A number of assumptions of the present model (mean–variance utility functions, quadratic cost functions, homogeneous agents) have allowed explicit computations (in particular of equilibria). These assumptions have made our analysis both simpler and more complete. To which extent would a relaxation of such assumptions affect our main results? The most likely answer is that a global analysis of eductive stability would either be impossible or much more difficult and that the reflection would have to focus on local stability tests – the success of which would make the equilibrium 'locally strongly rational' or 'locally eductively stable'. The present findings are then very likely to have general, but only local, counterparts.

5.5. Stability and the size of strategy spaces

One may conjecture that our main result has in fact nothing to do with futures markets and is a very peculiar case of a general game theoretical principle, namely that enlarging strategy spaces tend to destabilize Nash equilibria. This 'principle' is incorrect, for any meaning of the word 'destabilize'. For instance, when strategy spaces are enlarged, previously dominated strategies can become undominated and vice versa. Even for quadratic games

\[15\text{This local viewpoint is sometimes advocated and adopted, in Guesnerie (1988). Note that our stability criterion would then have some common features with a criterion often used for studying stability of infinite horizon equilibria and called Expectational Stability [see Evans and Guesnerie (1992) for a discussion of this point].}\]
like the ones studied here, one can find cases in which the Cournot tâtonnement becomes stable when strategy spaces are enlarged.

Therefore, there is some specific property of futures markets that is behind our Proposition 3. We believe it has to do with the 'redundancy' of such markets, in the sense that some players (the primary traders) already have the possibility to duplicate the outcome of futures markets by holding inventories. The only difference is that more players have access to this technology when the futures market is introduced, which is here destabilizing. Transposing our problems to options markets, we conjecture that similar destabilizing properties could be proved after an option market is created, but provided that this option is redundant in the classical sense of arbitrage pricing theory. That is, when markets are complete, introducing an option does not improve insurance possibilities, since all strategies involving this option could have been duplicated with already existing securities. The justification of creating such an option market has then to be related to market imperfections like borrowing constraints, shortsales restrictions and the like.

References

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